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Composite Vector and Tensor Gauge Fields, and Volume-Preserving Diffeomorphisms

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Abstract

We describe new theories of composite vector and tensor (*p-form*) gauge fields made out of zero-dimensional constituent scalar fields ("primitives"). The local gauge symmetry is replaced by an *infinite-dimensional global Noether symmetry* – the group of volume-preserving (symplectic) diffeomorphisms of the target space of the scalar primitives. We find additional non-Maxwell and non-Kalb-Ramond solutions describing topologically massive tensor gauge field configurations in odd space-time dimensions. Generalization to the supersymmetric case is also sketched.

1. Introduction Infinite-dimensional symmetries play an increasingly important rôle in various areas of physics. The current interest towards them is motivated mainly due to the recent discovery of effective description of Seiberg-Witten theory in terms of integrable models in lower space-time dimensions [1], as well as the rôle of $\mathbf{W_{1+\infty}}$-algebra in the field-theoretic description of the quantum Hall effect [2].

The hallmark of the completely integrable two-dimensional field-theoretic models is the presence of infinite sets of conservation laws allowing for their exact solvability. It is however difficult to find a realistic field theory in $D=4$ space-time dimensions possessing an infinite number of nontrivial conserved charges. In the present note (which is an extension of our work in ref.[3]) we will construct a series of new theories in higher space-time dimensions allowing for an infinite number of conservation laws, which at the same time closely resemble the standard vector and tensor (*p-form*) gauge theories. The basic idea is to introduce the ordinary gauge fields as composite fields built up of more elementary "primitive" scalar field constituents and to replace the ordinary local gauge symmetry with a *global infinite-dimensional* Noether symmetry acting on the target space of the scalar "primitives".

Let us briefly recall some basic notions connected with the infinite-dimensional groups $\mathrm{Diff}_0(\mathcal{T}^s)$ of volume-preserving diffeomorphisms on (s-dimensional) smooth manifolds $\mathcal{T}^s$. $\mathrm{Diff}_0(\mathcal{T}^s)$ is defined as the group of all diffeomorphisms preserving the canonical volume form $\frac{1}{s!}\varepsilon_{a_1\ldots a_s} d\varphi^{a_1}\wedge\ldots\wedge d\varphi^{a_s}$ (with $\{\varphi^a\}_{a=1}^s$ being a set of local coordinates) :

$$\mathrm{Diff}_0(\mathcal{T}^s) \equiv \left\{ \varphi^a \to G^a(\varphi)\; ;\; \varepsilon_{b_1\ldots b_s}\frac{\partial G^{b_1}}{\partial\varphi^{a_1}}\cdots\frac{\partial G^{b_s}}{\partial\varphi^{a_s}} = \varepsilon_{a_1\ldots a_s} \right\} \tag{1}$$

Accordingly, the Lie algebra $\mathcal{D}iff_0(\mathcal{T}^s)$ of infinitesimal volume-preserving diffeomorphisms is given by:

$$\mathcal{D}iff_0(\mathcal{T}^s) \equiv \left\{ \Gamma^a(\varphi) \;\; ; \;\; G^a(\varphi) \approx \varphi^a + \Gamma^a(\varphi) \; , \; \frac{\partial \Gamma^b}{\partial \varphi^b} = 0 \right\} \tag{2}$$

i.e.,

$$\Gamma^a(\varphi) = \frac{1}{(s-2)!} \varepsilon^{abc_1 \ldots c_{s-2}} \frac{\partial}{\partial \varphi^b} \Gamma_{c_1 \ldots c_{s-2}}(\varphi) \tag{3}$$

In the simplest case $s = 2$ the algebra $\mathcal{D}iff_0(\mathcal{T}^2)$ coincides with the algebra of symplectic (area-preserving) diffeomorphisms $\mathcal{SD}iff\,(\mathcal{T}^2) \equiv \left\{ \Gamma(\varphi) \; ; \; [\, \Gamma_1 \, , \, \Gamma_2 \,] \equiv \{\, \Gamma_1 \, , \, \Gamma_2 \,\} = \varepsilon^{ab} \frac{\partial \Gamma_1}{\partial \varphi^a} \frac{\partial \Gamma_2}{\partial \varphi^b} \right\}$ which contains as a subalgebra the centerless conformal Virasoro algebra and whose Lie-algebraic deformation is the well-known $\mathbf{W_{1+\infty}}$-algebra.

2. Composite p-form Gauge Theories Let us consider a set of $s(\equiv p+1)$ zero-dimensional scalar fields $\{\varphi^a(x)\}_{a=1}^s$ taking values in a smooth manifold $\mathcal{T}^s$. The pull-back of its canonical volume $s(\equiv p+1)$-form to Minkowski space-time gives rise to an antisymmetric s-tensor gauge field strength and its associated antisymmetric $(s-1)$-tensor gauge potential:

$$\frac{1}{s!} \varepsilon_{a_1 \ldots a_s} d\varphi^{a_1} \wedge \cdots \wedge d\varphi^{a_s} = \frac{1}{s!} F_{\mu_1 \ldots \mu_s}(\varphi) dx^{\mu_1} \wedge \cdots \wedge dx^{\mu_s} \tag{4}$$

$$F_{\mu_1 \ldots \mu_s}(\varphi) = \varepsilon_{a_1 \ldots a_s} \, \partial_{\mu_1} \varphi^{a_1} \ldots \partial_{\mu_s} \varphi^{a_s} \tag{5}$$

$$F_{\mu_1 \ldots \mu_s}(\varphi) = s \partial_{[\mu_1} A_{\mu_2 \ldots \mu_s]}(\varphi) \quad , \quad A_{\mu_1 \ldots \mu_{s-1}}(\varphi) = \frac{1}{s} \varepsilon_{a_1 \ldots a_s} \varphi^{a_1} \partial_{\mu_1} \varphi^{a_2} \ldots \partial_{\mu_{s-1}} \varphi^{a_s} \tag{6}$$

where the square brackets indicate total antisymmetrization of indices. One can easily verify that the field strength (5) is invariant under arbitrary field transformations (reparametrizations) $\varphi^a(x) \to G^a\,(\varphi(x))$ belonging to the infinite-dimensional group $\mathrm{Diff}_0(\mathcal{T}^s)$ (1), whereas its potential (6) undergoes a φ-dependent $(s-2)$-rank local gauge transformation (cf. (2)–(3)):

$$A_{\mu_1 \ldots \mu_{s-1}}(\varphi) \to A_{\mu_1 \ldots \mu_{s-1}}(\varphi) + (s-1) \partial_{[\mu_1} \Lambda_{\mu_2 \ldots \mu_{s-1}]}(\varphi) \tag{7}$$

$$\Lambda_{\mu_1 \ldots \mu_{s-2}}(\varphi) = \left[\left(1 - \frac{1}{s} \varphi^b \frac{\partial}{\partial \varphi^b} \right) \Gamma_{a_1 \ldots a_{s-2}}(\varphi) + \frac{(s-2)}{s} \varphi^b \frac{\partial}{\partial \varphi^{[a_1}} \Gamma_{|b|a_2 \ldots a_{s-2}]} \right] \times \\ \times \, \partial_{\mu_1} \varphi^{a_1} \ldots \partial_{\mu_{s-2}} \varphi^{a_{s-2}} \tag{8}$$

In the simplest case $s\!=\!2$ the composite electromagnetic field strenght and potential read:

$$F_{\mu\nu}(\varphi) = \varepsilon_{ab} \partial_\mu \varphi^a \partial_\nu \varphi^b \quad , \quad F_{\mu\nu}(\varphi) = \partial_\mu A_\nu(\varphi) - \partial_\nu A_\mu(\varphi) \tag{9}$$

$$A_\mu(\varphi) = \frac{1}{2} \varepsilon_{ab} \varphi^a \partial_\mu \varphi^b \quad , \quad A_\mu(\varphi) \to A_\mu(\varphi) + \partial_\mu \left(\Gamma(\varphi) - \frac{1}{2} \varphi^a \frac{\partial \Gamma}{\partial \varphi^a} \right) \tag{10}$$

Now it is straightforward, upon using (5)–(6), to construct field-theory models of arbitrary *p-form* tensor gauge fields involving the scalar "primitives" φ^a coupled to ordinary matter (*e.g.*, fermionic) fields, where the standard local *p-form* tensor gauge invariance is substituted with the *global infinite-dimensional* Noether symmetry of volume-preserving diffeomorphisms on the $s \equiv p+1$-dimensional target space of primitive scalar constituents. The simplest model (for $s = 2$) is the so called "mini-QED" [3] : $\mathcal{L} = -\frac{1}{4e^2} F^2_{\mu\nu}(\varphi) +$

$\bar{\psi}\left(i\partial\!\!\!/ - A\!\!\!/(\varphi) - im\right)\psi$, where $F_{\mu\nu}(\varphi)$ and $A_\mu(\varphi)$ are given by (9) and (10), respectively. Similarly, the arbitrary higher rank *p-form* composite tensor gauge field theories are defined by:

$$S = -\frac{1}{2(p+1)e^2}\int d^D x\, F^2_{\mu_1\ldots\mu_{p+1}}(\varphi(x)) + \int d^D x\, A_{\mu_1\ldots\mu_p}(\varphi(x))\, J^{\mu_1\ldots\mu_p}_{matter}(x) + S_{matter} \quad (11)$$

where $F_{\mu_1\ldots\mu_{p+1}}(\varphi(x))$ and $A_{\mu_1\ldots\mu_p}(\varphi(x))$ are given by (5)–(6). In particular, for $s=3$ we have a "mini-Kalb-Ramond" model:

$$\mathcal{S} = -\frac{1}{3!e^2}\int d^D x\, F^2_{\mu\nu\lambda}(\varphi(x)) + \int d^2\sigma\, \frac{1}{3}\varepsilon_{abc}\,\varphi^a(x(\sigma))\partial_{\sigma_1}\varphi^b(x(\sigma))\partial_{\sigma_2}\varphi^c(x(\sigma)) \quad (12)$$

where the second integral is over the string world-sheet given by $x^\mu = x^\mu(\sigma)$.

Let us particularly stress that, although the Lagrangians of the composite *p-form* gauge theories (11)–(12) contain higher order derivatives w.r.t. φ^a, they are only quadratic w.r.t. time-derivatives. Also, note that the Chern-Simmons terms and "topolocal" densities for the composite *p-form* tensor gauge fields (5)–(6) *identically* vanish in space-time dimensions $D=2p+1$ and $D=2p+2$, respectively, *e.g.* :

$$\varepsilon^{\mu_1\ldots\mu_{2p+1}} A_{\mu_1\ldots\mu_p}(\varphi)F_{\mu_{p+1}\ldots\mu_{2p+1}}(\varphi) =$$

$$\frac{1}{p+1}\varepsilon_{a_1\ldots a_{p+1}}\,\varepsilon_{b_1\ldots b_{p+1}}\,\varepsilon^{\mu_1\ldots\mu_{2p+1}}\varphi^{a_1}\partial_{\mu_1}\varphi^{[a_2}\ldots\partial_{\mu_p}\varphi^{a_{p+1}}\partial_{\mu_{p+1}}\varphi^{b_1}\ldots\partial_{\mu_{2p+1}}\varphi^{b_{p+1}]} = 0 \quad (13)$$

due to the total antisymmetrization of the $2p+1$ indices $a_2,\ldots,a_{p+1},b_1,\ldots,b_{p+1}$ taking only $p+1$ values.

3. Non-Maxwell and Non-Kalb-Ramond Solutions The action (11) yields the following equations of motion: $\varepsilon_{ab_1\ldots b_p}\partial_{\mu_1}\varphi^{b_1}\ldots\partial_{\mu_p}\varphi^{b_p}\left[\frac{1}{e^2}\partial_\nu F^{\nu\mu_1\ldots\mu_p}(\varphi(x)) + J^{\mu_1\ldots\mu_p}_{matter}(x)\right] = 0$ upon variation w.r.t. φ^a or, equivalently, upon multiplying the l.h.s. of the last equation by φ^a and accouting for (6) :

$$A_{\mu_1\ldots\mu_p}(\varphi(x))\left[\frac{1}{e^2}\partial_\nu F^{\nu\mu_1\ldots\mu_p}(\varphi(x)) + J^{\mu_1\ldots\mu_p}_{matter}(x)\right] = 0 \quad (14)$$

From (14) we notice that any solution $A_{\mu_1\ldots\mu_p}(x)$ of the standard *p-form* gauge theory, *i.e.*, such $A_{\mu_1\ldots\mu_p}(x)$ for which the term in the square brackets in (14) vanishes, is authomatically a solution of the new composite *p-form* gauge theory (11) provided $A_{\mu_1\ldots\mu_p}(x)$ is representable (up to $(p-1)$-rank gauge transformation) in terms of scalar "primitives" as in (6).

On the other hand, Eqs.(14) possess additional solutions unattainable in ordinary *p-form* gauge theories, namely such $A_{\mu_1\ldots\mu_p}(\varphi(x))$ for which the corresponding factor in the square brackets in (14) is non-zero:

$$\frac{1}{e^2}\partial_\nu F^{\nu\mu_1\ldots\mu_p}(\varphi(x)) + J^{\mu_1\ldots\mu_p}_{matter}(x) + \mathcal{J}^{\mu_1\ldots\mu_p}(\varphi(x)) = 0 \quad (15)$$

where the additional "current" $\mathcal{J}^{\mu_1\ldots\mu_p}$ obeys: $A_{\mu_1\ldots\mu_p}(\varphi(x))\,\mathcal{J}^{\mu_1\ldots\mu_p}(\varphi(x)) = 0$. An immediate nontrivial example for $\mathcal{J}^{\mu_1\ldots\mu_p}$ satisfying the latter constraint in odd space-time dimensions $D=2p+1$ is (due to (13)) :

$$\mathcal{J}^{\mu_1\ldots\mu_p}(\varphi(x)) = \frac{1}{(p+1)!}\varepsilon^{\mu_1\ldots\mu_p\nu_1\ldots\nu_{p+1}}F_{\nu_1\ldots\nu_{p+1}}(\varphi(x)) \quad (16)$$

so that Eqs.(15) become the eqs. of motion of ordinary *p-form* gauge theory with *additional* Chern-Simmons term. Thus, if we succeed to find a solution to:

$$\frac{1}{e^2}\partial_\nu F^{\nu\mu_1\ldots\mu_p} + \frac{1}{(p+1)!}\varepsilon^{\mu_1\ldots\mu_p\nu_1\ldots\nu_{p+1}} F_{\nu_1\ldots\nu_{p+1}} + J^{\mu_1\ldots\mu_p}_{matter} = 0 \tag{17}$$

which is expressible in terms of scalar "primitives", we therefore obtain a non-Maxwell/non-Kalb-Ramond solution of Eqs.(14).

We will present here such a solution of Eqs.(14) in the free case $J^{\mu_1\ldots\mu_p}_{matter}(x) = 0$ (for simplicity, we consider the "mini-Kalb-Ramond" model (12)) :

$$A^{\mu\nu}(\varphi)\,\partial^\lambda F_{\lambda\mu\nu}(\varphi) = 0\ ,\ \ \partial^\lambda F_{\lambda\mu\nu}(\varphi) + \frac{e^2}{3!}\varepsilon_{\mu\nu\rho\sigma\tau}F^{\rho\sigma\tau}(\varphi) = 0\ ,\ \ F_{\lambda\mu\nu}(\varphi) = \varepsilon_{abc}\partial_\lambda\varphi^a\partial_\mu\varphi^b\partial_\nu\varphi^c \tag{18}$$

$$\varphi^1(x) = k_\mu x^\mu \quad , \quad \varphi^2(x) = \ell_\mu x^\mu \quad , \quad \varphi^3(x) = x_\mu\left[p^\mu \sin(k.x) + q^\mu \cos(k.x)\right] \tag{19}$$

where k, ℓ, p, q are constant 5-vectors obeying:

$$\begin{gathered} p.k = p.\ell = q.k = q.\ell = k.\ell = p.q = \ell^2 = 0 \quad , \quad p^2 = q^2 \\ k^2\left(p_\mu\ell_\nu - p_\nu\ell_\mu\right) - e^2\varepsilon_{\mu\nu\rho\sigma\tau}k^\rho\ell^\sigma q^\tau = 0 \quad , \quad k^2\left(q_\mu\ell_\nu - q_\nu\ell_\mu\right) + e^2\varepsilon_{\mu\nu\rho\sigma\tau}k^\rho\ell^\sigma p^\tau = 0 \end{gathered} \tag{20}$$

Hence, we find from Eqs.(18)–(19) that composite *p-form* tensor gauge theories (11) exhibit dynamical parity breakdown since they carry "topologically" massive excitations.

4. Supersymmetric Generalization Here we will briefly indicate the supersymmetric generalization of (11) in the simplest case of interest: $s=2$ ("mini-super-QED") and $D=4$ (for superspace notations and definitions, see *e.g.* [4]). The superfield of the vector supermultiplet $(A_\mu, \lambda_\alpha, \bar\lambda^{\dot\alpha})$:

$$\begin{aligned} V(x,\theta,\bar\theta) &= a(x) + \theta_\alpha\chi^\alpha(x) + \bar\theta_{\dot\alpha}\bar\chi^{\dot\alpha}(x) + \delta(\theta)\bar b(x) - \delta(\bar\theta)b(x) + (\theta\sigma^\mu\bar\theta)A_\mu(x) + \\ &+ \delta(\bar\theta)\theta_\alpha(\lambda^\alpha(x) + i\partial^{\alpha\dot\beta}\bar\chi_{\dot\beta}(x)) - \delta(\theta)\bar\theta_{\dot\alpha}(\bar\lambda^{\dot\alpha}(x) + i\partial^{\dot\alpha\beta}\chi_\beta(x)) + \delta(\theta)\delta(\bar\theta)(\mathcal{D}(x) - \partial^2 a(x)) \end{aligned} \tag{21}$$

(here $a, b, \bar b, \chi^\alpha, \bar\chi^{\dot\alpha}$ and $\mathcal{D}$ are pure-gauge and auxiliary component fields, respectively, and $\delta(\theta) \equiv \frac{1}{2}\theta_\alpha\theta^\alpha$, $\delta(\bar\theta) \equiv \frac{1}{2}\bar\theta^{\dot\alpha}\,\bar\theta_{\dot\alpha}$) is constructed as bilinear composite in terms of a pair of chiral and anti-chiral "primitive" scalar superfields:

$$\Phi(x,\theta,\bar\theta) = e^{-i(\theta\sigma^\mu\bar\theta)\partial_\mu}\left[\varphi(x) + \theta_\alpha\psi^\alpha + \delta(\theta)f(x)\right] \tag{22}$$

$$\bar\Phi(x,\theta,\bar\theta) = e^{i(\theta\sigma^\mu\bar\theta)\partial_\mu}\left[\bar\varphi(x) + \bar\theta_{\dot\alpha}\bar\psi^{\dot\alpha} - \delta(\bar\theta)\bar f(x)\right] \tag{23}$$

through the following simple formula:

$$V(\Phi,\bar\Phi) = \Phi\,\bar\Phi \tag{24}$$

In terms of component fields Eq.(24) reads:

$$A_\mu = -i\bar\varphi\overleftrightarrow{\partial}_\mu\varphi + \bar\psi^{\dot\alpha}(\sigma_\mu)_{\alpha\dot\alpha}\psi^\alpha \quad , \quad \lambda^\alpha = -2i\bar\psi_{\dot\beta}\partial^{\dot\beta\alpha}\varphi - f\psi^\alpha \quad , \quad \bar\lambda^{\dot\alpha} = -2i\psi_\beta\partial^{\dot\alpha\beta}\bar\varphi - \bar f\bar\psi^{\dot\alpha} \tag{25}$$

which are the supersymmetric generalization of (10). The superspace action of "mini-super-QED" is obtained by replacing the superfield V (21) with its composite form (24) in the well-known super-QED action:

$$S = -\frac{1}{16e^2} \int d^4x \, d^2\theta \, W^{\alpha}(\Phi, \bar{\Phi}) \, W_{\alpha}(\Phi, \bar{\Phi}) + \int d^4x \, d^2\theta \, d^2\bar{\theta} \, \bar{\mathcal{M}} \, e^{V(\Phi, \bar{\Phi})} \, \mathcal{M} \tag{26}$$

where $W_\alpha = \frac{1}{2}\bar{D}^{\dot\beta}\bar{D}_{\dot\beta} D_\alpha V$ is the supersymmetric field strenght (with $D^\alpha = \frac{\partial}{\partial\theta_\alpha} - i\partial^{\alpha\dot\beta}\bar{\theta}_{\dot\beta}$ and $\bar{D}_{\dot\alpha} = \frac{\partial}{\partial\bar{\theta}^{\dot\alpha}} - i\partial_{\beta\dot\alpha}\theta^\beta$ being the standard super-derivatives) and $\mathcal{M}, \bar{\mathcal{M}}$ are the chiral/antichiral matter superfields.

5. Outlook It is an interesting task to study in detail the supersymmetric composite *p-form* gauge theories generalizing (26). Of particular interest to modern string theory would be to find supersymmetric p-brane solutions in these theories extending the work in ref.[5], where p'-brane solutions have been obtained in the purely bosonic composite *p-form* tensor gauge theories (11) when $D = (p+1) + (p'+1)$. On the other hand, the possibility of defining volume forms in terms of "primitive" scalar fields as in (9) suggests the idea of replacement of the standard measure of integration $\sqrt{-g}$ in general-coordinate invariant theories by the "primitive" scalar composite $\det \|\partial_\mu \varphi^a\|$ (here $p+1=D$). The basic reasons and advantages of this approach are discussed in separate contributions to these Proceedings (by E.I. Guendelman and A.B. Kaganovich and by E.I.Guendelman)).

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